

# Isolating the Dynamic Dipolar Interaction between a Pair of Nanoscale Ferromagnetic Disks

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Dynamic dipolar interactions between spin wave eigenmodes of closely spaced nanomagnets determine the collective behavior of magnonic and spintronic metamaterials and devices. However, dynamic dipolar interactions are difficult to quantify since their effects must be disentangled from those of static dipolar interactions and variations in the shape, size, and magnetic properties of the nanomagnets. It is shown that when two imperfect nanoscale magnetic disks with similar but nonidentical modes are brought into close proximity, the effect of the dynamic dipolar interaction can be detected by considering the difference of the phase of precession within the two disks. Measurements show that the interaction is stronger than expected from micromagnetic simulations, highlighting both the need for characterization and control of magnetic properties at the deep nanoscale, and also the potential for improved control of collective magnetic phenomena. Our approach is equally applicable to other physical systems in which dynamic interactions are obscured by inhomogeneous broadening and static interactions.

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The rapid development of spintronics [1] and magnonics [2] is fueled by the potential for energy efficient technologies that exploit the spin rather than the charge of the electron. Recently spin transfer oscillators (STOs) have been proposed as highly agile components for microwave signal processing, with bottleneck issues of emitted power level and frequency linewidth being overcome by non-linear phase locking of multiple STOs [3,4]. On the other hand, two-dimensional (2D) magnonic crystals have been found to exhibit bands of allowed magnonic states and frequency-forbidden magnonic band gaps [5,6] that may find potential applications in magnonic filters and logic devices. While the technological applications of these multicomponent nanoscale systems may differ, their operation relies on dynamical coupling between resonances within the individual nanomagnets. Nanomagnet systems present an excellent opportunity to study dynamic coupling phenomena since their interaction can be easily tuned by an applied magnetic field [7].

Coupled nanomagnet arrays are examples of a wider class of metamaterials in which bands of collective modes result from dynamic interactions between constituent elements. At GHz frequencies collective magnonic modes occur in 2D arrays of nanomagnets due to dynamic dipolar coupling [5,6,8], phononic modes occur in nonmagnetic metallic dot arrays due to elastic interactions mediated by the substrate [9], and the precession of a spin within a quantum dot is modified by interdot tunneling used to produce entangled states for quantum computation [10].

At THz frequencies, collective plasmonic excitations in concentric metallic nanoshells [11] and self-assembled metallic colloidal clusters [12] result from electric dipole interactions. In each case it is necessary to understand the effect of dynamic interactions to control the collective properties. In this work we demonstrate a method to isolate the dynamic interaction by considering the difference in phase of the individual resonances.

Recent studies have led to an improved understanding of the spin wave eigenmodes of individual confined magnetic structures [13–17], while the most effective means of coupling the modes of closely spaced structures is less clear [3,5,6,18–20]. The magnetic dipolar interaction present in any magnetic system is an obvious candidate. The precessing magnetization of a confined magnetic structure generates a time varying magnetic field at a neighboring structure that couples their precession with a well-defined relative phase, forming collective modes [5,6,8,21].

The dynamic response of an in-plane magnetized planar nanomagnet is characterized by a center mode with large amplitude at the center, and edge modes with large amplitude in demagnetized regions at the edges perpendicular to the applied magnetic field [13,15,16,22,23]. Within arrays of nanomagnets the effect of dipolar interactions is difficult to identify due to inhomogeneous broadening, particularly of edge modes, associated with structural imperfections [16,23]. The number of collective modes within an array is the product of the number of nanomagnets and the number

of eigenmodes of each nanomagnet. Therefore, the effect of interactions is most easily isolated by measuring a single pair of nanomagnets. In this letter we demonstrate that the effect of dynamic dipolar interactions between center modes, and between edge modes, of two in-plane magnetized nanomagnets can be isolated by separately measuring the phase-resolved response of each nanomagnet.

Time-resolved scanning Kerr microscopy (TRSKM) with coherent sinusoidal microwave (RF) excitation [24–26] of the sample has been used to study a pair of single domain ferromagnetic nano-disks interacting via dipolar interactions. Pairs of disks with nominal diameter  $d$  of 300 nm were fabricated using electron-beam lithography and ion-beam milling from a sputtered Cu(6 nm)/Ni<sub>80</sub>Fe<sub>20</sub>(15 nm)/Cu(3 nm)/Al<sub>2</sub>O<sub>3</sub>(2 nm) continuous film stack deposited on a 0.5 mm thick Si/SiO<sub>2</sub> substrate. The nominal edge-to-edge separation  $s$  ranged from 600 nm to 90 nm corresponding to  $s/d$  values of 2, 1, 0.6, and 0.3.

The disks were fabricated at the center of a microwave antenna that was used to generate an out-of-plane RF magnetic field  $\mathbf{h}_{\text{RF}}(t)$  with amplitude less than 20 Oe, Fig. 1(a). The disks were magnetized in-plane by a magnetic field  $\mathbf{H}$  applied either parallel or perpendicular to the center-to-center direction of the pair, in what will be referred to as the parallel and perpendicular geometries. The RF field was synchronized with the arrival of  $\sim 100$  fs laser pulses that were used to stroboscopically detect the

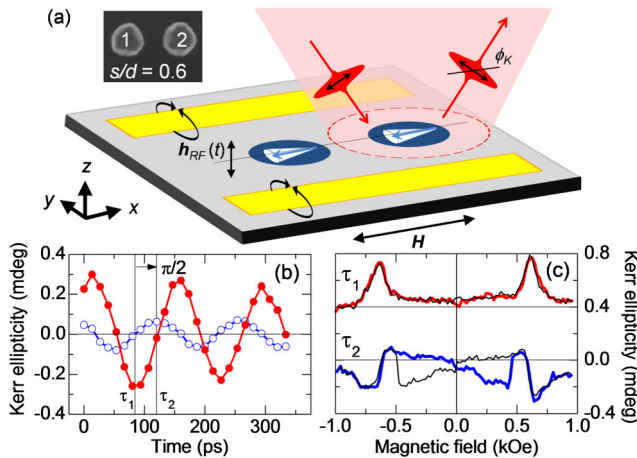


FIG. 1 (color online). (a) A schematic of the experiment and a scanning electron microscope (SEM) image (inset) of a pair of 300 nm disks with  $s/d = 0.6$ . (b) TR signals acquired at 7.2 GHz from disk 2 in the perpendicular geometry at remanence (blue curve, open symbols) and at the resonance field ( $H_R = -640$  Oe) of the center mode (red curve, closed symbols). (c)  $\text{Im}(\chi_{zz})$  and  $\text{Re}(\chi_{zz})$  curves acquired at time delays corresponding to the node ( $\tau_1$ ) and antinode ( $\tau_2$ ), respectively, of the signal acquired at remanence in (b). The susceptibility curves were acquired by sweeping the applied field from  $-1$  to  $+1$  kOe (thick colored curves), and then from  $+1$  to  $-1$  kOe (thin black curves) to complete a single hysteresis cycle.

out-of-plane component of the dynamic magnetization  $M_z$  using the polar magneto-optical Kerr effect  $\phi_K$ . The probe had 800 nm wavelength and was focused to a  $\sim 500$  nm diffraction limited spot diameter by a  $\times 60$  microscope objective (0.85 numerical aperture). The high spatial resolution allowed the magnetization dynamics of each disk to be measured independently. To acquire time resolved (TR) signals, a 4 ns optical delay line was used to control the phase of the probe pulse with respect to the RF field. Alternatively, the TRSKM was used to perform phase-resolved ferromagnetic resonance measurements [24], in which the applied field was swept through  $\pm 1$  kOe, while the phase was kept fixed, as illustrated in Fig. 1.

In Fig. 1(b) a TR signal resulting from the center mode on resonance at 7.2 GHz and  $-640$  Oe is shown. At remanence (18 Oe) the phase of the TR signal is shifted by  $\pi/2$  radians and the amplitude is reduced, as expected for a driven damped harmonic oscillator when the driving frequency is increased from the resonant value to a much larger value. By setting the time delay to that of a node  $\tau_1$  or an antinode  $\tau_2$  of the signal observed at remanence, curves corresponding to the imaginary and real parts of the magnetic susceptibility tensor component  $\chi_{zz}$  are obtained [24] [ $\text{Im}(\chi_{zz})$  and  $\text{Re}(\chi_{zz})$  in Fig. 1(c)]. From saturation at  $-1$  kOe to remanence the disks have a quasiuniform single domain ground state [26]. In Fig. 1(c) the absorptive  $\text{Im}(\chi_{zz})$  and dispersive  $\text{Re}(\chi_{zz})$  type resonance curves for the center mode are seen to be centered at  $-640$  Oe. The disks have a vortex ground state from remanence to the vortex annihilation field at  $\sim 500$  Oe, and a continuum of modes associated with a displaced vortex is observed in the  $\text{Re}(\chi_{zz})$  spectrum [17]. Above  $\sim 500$  Oe the disks return to the single domain state. Here we discuss dynamic dipolar coupling of center and edge modes of the single domain state only.

In Fig. 2 the  $s/d$  dependence of center and edge mode resonance fields at 4.4 GHz is shown for the parallel (a) and perpendicular (b) geometry. At 4.4 GHz both center and edge modes can be excited. The range of resonance fields for the center and edge modes is larger for the parallel geometry, while the center mode shows a smaller range of values compared to the edge mode in both geometries since the center mode is relatively “isolated” from dipolar interactions and structural distortions of the disks. In the parallel geometry the internal magnetic field of the disks is asymmetric along the applied field direction due to the static dipolar field generated by each disk. Micromagnetic simulations show that the asymmetry modifies the spatial character of the center mode, and strongly splits the degenerate edge modes of an ideal single disk [27]. Consequently, the dynamic dipolar interaction field is expected to exhibit a strong dependence upon the value of  $s/d$ .

In the perpendicular geometry for  $s/d = 0.3$  one of the edge modes observed in each disk (triangles) lies

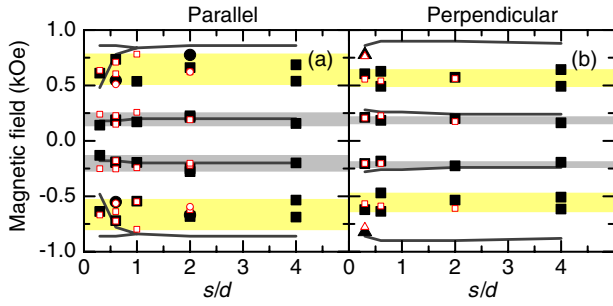


FIG. 2 (color online). Center and edge mode resonance fields of pairs of disks with different  $s/d$  at 4.4 GHz for parallel (a) and perpendicular (b) geometries. Single disk data are shown at  $s/d = 4$ , while disk 1 and disk 2 are shown as closed black and open red symbols. Simulated resonance fields are shown as gray curves. In (a) the circles denote measurements from a second pair of disks at  $s/d = 0.6$  and 2. The range of resonance fields for center and edge modes are shown as gray and yellow bands. Outlying resonance fields shown as triangles for  $s/d = 0.3$  in (b) are discussed in the main text.

outside the field range observed for larger  $s/d$  values. Micromagnetic simulations at 500 Oe show that the center mode is symmetric while the edge modes are degenerate in field since the internal field is symmetric along the applied field direction in the perpendicular case. Therefore, strong splitting of the edge mode for  $s/d = 0.3$  is most likely the result of structural or magnetic imperfections. While SEM images of the disks reveal that their shape is not ideal [e.g., Fig. 1(a) inset], the structural distortions of the disks are at least similar within each pair, leading to similar mode splitting in disk 1 (d1) and disk 2 (d2).

In Fig. 2 micromagnetic simulations show the expected resonance fields as a function of  $s/d$ . The variation of the measured resonance fields due to the effects of shape, size, and static dipolar interactions demonstrates that it is not straightforward to observe the effect of dynamic interactions. Comparison of simulations of a pair of disks with the magnetization of one disk either fixed or free to precess revealed that dynamic dipolar fields generated by center and edge modes of one disk have little effect on the mode frequencies of the other disk, but were found to significantly change their phase.

Simulations of two parallel macrospins, with slightly different resonance fields (as in the experiment), were used to illustrate how their phase can be used to isolate the effect of the dynamic dipolar interaction [28]. In the absence of dynamic coupling, the difference in the phase of two nonidentical macrospins resulted in a “phase-difference peak” at the center of the resonances. In the presence of dynamic coupling the phase-difference peak was found to shift from the center of the resonances. The sign of the shift depends on the sign of the coupling, while the magnitude of the shift depends on the strength of the dynamic interaction. In fact, for any pair of nonidentical coupled oscillators the dynamic interaction leads to a

shift in the phase difference peak. The same analysis of the phase difference was applied to micromagnetic simulations of a pair of ideally shaped disks with slightly different diameters (300 and 275 nm) and  $s/d \sim 0.6$  [29]. Different diameters led to slightly different resonance fields, as in the experiment. The phase difference peak for the center and edge modes (inward facing edges) was found to shift to larger values of field by 6 and 14 Oe, respectively. The larger shift for the edge modes demonstrates stronger dynamic dipolar coupling due to the closer proximity and larger precession angle of the edge modes.

Experimentally, the amplitude and phase for each disk can be either directly extracted from TR signals or calculated from the  $\text{Re}(\chi_{zz})$  and  $\text{Im}(\chi_{zz})$  curves [30] as  $\sqrt{\text{Re}(\chi_{zz})^2 + \text{Im}(\chi_{zz})^2}$  and  $\arctan(\text{Re}(\chi_{zz})/\text{Im}(\chi_{zz}))$  respectively. In Fig. 3 the signal amplitude for each disk within a pair is shown in the parallel geometry at 4.4 GHz for  $s/d$  values of 2 (a), 1 (b), 0.6 (c) and 0.3 (d), along with the difference of the phase curves obtained from the two disks. For  $s/d = 2$  each disk exhibits a lower field center mode and a higher field edge mode, as expected from micromagnetic simulations. The resonance field for each mode is slightly different, due to the slightly different shape, size, and magnetic parameters of each disk [26]. Since  $s/d$  is large, static and dynamic dipolar interactions

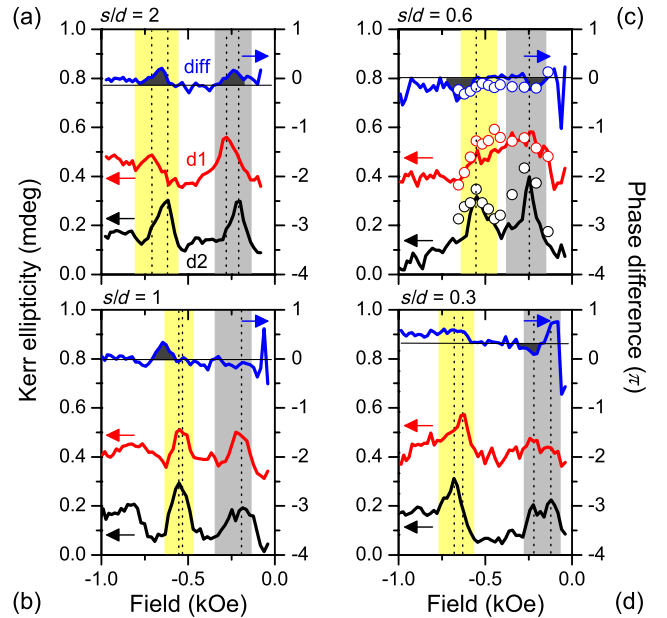


FIG. 3 (color online). Measured amplitude for disk 1 (d1, red curves) and disk 2 (d2, black curves) and phase difference (diff, blue curves), are shown for pairs of disks in the parallel geometry and at 4.4 GHz with separation corresponding to  $s/d = 2, 1, 0.6,$  and  $0.3$  in (a), (b), (c), and (d), respectively. In (c) the open symbols correspond to the amplitude and phase difference extracted by fitting the TR signals acquired from disk 1 and disk 2. Center and edge modes appear within the gray and yellow bands, respectively.

are weak and strong splitting of the edge mode due to the nonuniform static field is not observed. For a particular mode the phase difference peak (dark gray shading) lies between the resonances observed in the two disks, confirming that the dynamic interaction is weak or absent.

For  $s/d = 1$ , and 0.6, the edge mode resonance field decreases due to the static field of the neighboring disk. For  $s/d = 1$ , center and edge modes are resolved. For  $s/d = 0.6$ , d2 again shows clearly resolved modes, but d1 shows two unresolved modes within a broad peak of large amplitude around  $-250$  Oe, and a shoulder around  $-500$  Oe. For  $s/d = 1$ , a second edge mode may be present in each disk around  $-750$  Oe due to splitting resulting from structural distortions [26]. Phase difference curves for  $s/d$  of 1 and 0.6 show a substantial peak that is shifted to values of magnetic field larger than that of the associated edge mode resonances, in agreement with micromagnetic simulations. The difference in sign of the phase difference curves for  $s/d = 1$  and 0.6 is ascribed simply to differences in the disks within each pair, which dictates in which disk the precession will lag that of the other. As  $s/d$  decreases, the shift of the phase difference peak with respect to the average value of the edge mode resonance field increases monotonically. For the center mode, the phase difference peak is not always clear. For  $s/d = 1$  the center modes have very similar resonance fields, so a phase difference peak is not expected, or observed. However, for  $s/d = 0.6$ , a small peak is observed, suggesting that for smaller  $s/d$  values the center modes begin to interact dynamically as suggested by micromagnetic simulations.

For  $s/d = 0.6$ , the amplitude and phase difference were also extracted from TR signals acquired from each disk at different values of the applied magnetic field. TR signals acquired at  $-620$  Oe (not shown) reveal a  $37.4$  ps phase difference ( $0.32\pi$  at  $4.4$  GHz) between the edge modes of d1 and d2 that is clearly resolved and in reasonable agreement with the amplitude of the phase difference peak calculated from the measured  $\text{Re}(\chi_{zz})$  and  $\text{Im}(\chi_{zz})$  curves. However, the smaller phase difference peak of the center mode is not well reproduced by the TR signals due to the limited phase resolution of the experiment [31].

For  $s/d = 0.3$  the amplitude spectra show the higher field edge mode of each disk, and center modes that have been split into at least two modes with reduced amplitude, in agreement with micromagnetic simulations. When the average simulated  $M_z$  response of each disk is considered, the amplitude of the center mode is always larger than that of the edge mode due to the larger center mode volume and smaller edge mode volume. Edge modes with similar amplitude to the center mode are routinely observed in Fig. 3, suggesting that the volume occupied by and/or the amplitude of the edge mode is substantially larger than expected from micromagnetic simulations. It follows that the dynamic dipolar interaction (i.e., shift of the phase difference peak) between edge modes cannot easily be

reproduced by micromagnetic simulations that assume a uniform profile of magnetic parameters and no shape distortions. For  $s/d = 0.3$  the phase difference spectrum is very different from those observed for larger  $s/d$ . A strong phase variation is observed near to the edge mode resonance of d1, while at larger values of the applied magnetic field the phase difference remains almost constant. That is, the precession of the edge modes may be phase locked by their strong dynamic dipolar interaction for a range of applied field values, despite their slightly different resonance field values.

Both the measured linewidth and the shift of the phase difference peak are larger than expected from micromagnetic simulations. To reproduce the observed linewidth it is necessary to increase the Gilbert damping parameter from 0.01 to 0.03, which suggests increased 2-magnon scattering from edge imperfections [32]. The size of the shift of the phase difference peak is directly related to the strength of the dynamic dipolar coupling. The observed shift requires the coupling parameter [28] within the macrospin model to be  $\times 10$  larger than estimated for a pair of point dipoles located at the position of maximum mode amplitude within each disk. The enhanced coupling may also be a consequence of the nanofabrication process; e.g., ion milling leads to a reduction of the exchange parameter at the edges [33], leading to increased edge mode amplitude. The large edge mode amplitude observed experimentally lends support to this idea. Modified magnetic parameters may also modify the spatial profile of the center mode with enhanced dynamic coupling mediated by increased amplitude near edges of the disks. Enhanced dynamic dipolar coupling is potentially of great importance in controlling systems that contain multiple nanomagnets and the results presented here may stimulate further work on the fabrication and characterization of nanomagnets with graded magnetic profiles.

In summary, we have demonstrated that the weak dynamic dipolar interaction can be isolated from the stronger static interaction and the effects of structural and magnetic imperfections by measuring the phase of center and edge modes of each disk within a pair. This method may be applied to other coupling mechanisms, e.g., flow of spin current, or to any system of nonidentical coupled oscillators. Our results confirm that the dynamic coupling depends strongly upon the disk separation and applied magnetic field, and that a judicious choice of center or edge mode excitation may be required if dipolar interactions are to deliver tailored collective modes or nonlinear phase locking of STOs. The observation of a stronger dynamic interaction than expected from simulations gives impetus to deep nanoscale characterization of local magnetic properties and suggests that enhanced dynamic dipolar coupling may be achieved by engineering the spatial variation of magnetic properties, particularly within edge regions.

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- [29] See Supplemental information at <http://link.aps.org/supplemental/10.1103/PhysRevLett.110.187202> for details of the micromagnetic simulations.
- [30] A constant background was subtracted from measured spectra that had suffered an offset due to inductive pickup of the lock-in amplifier audio frequency reference waveform.
- [31] This is due to a limitation of the experiment, whereby mechanical drift at large static field values and phase slips of the microwave synthesizer are more pronounced over the time scales of the TR measurements than those of the field swept spectra. While every effort was made to ensure that these effects were minimized, the smallest phase difference peaks extracted from the TR signals may be unresolved.
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