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Synchronization of spin-transfer oscillators driven by stimulated microwave currents

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We have simulated the nonlinear dynamics of networks of spin-transfer oscillators. The oscillators are magnetically uncoupled but electrically connected in series. We use a modified Landau-Lifschitz-Gilbert equation to describe the motion of each oscillator in the presence of the oscillations of all the others. We show that the oscillators of the network can be locked not only in frequency but also in phase. The coupling is due to the microwave components of the current induced in each oscillator by the oscillations in all the other oscillators. Our results show how the emitted microwave power of spin-transfer oscillators can be considerably enhanced by current-induced synchronization in an electrically connected network. We also discuss the possible application of our synchronization mechanism to the interpretation of the surprisingly narrow microwave spectrum in some experiments on a single isolated spin-transfer oscillator.

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The spin-transfer phenomenon, predicted by Slonczewski[1] in 1996, is now the subject of extensive experimental[2–9] and theoretical studies.[10–15] It has first been shown that a spin-polarized current injected into a thin ferromagnetic layer can switch its magnetization. This occurs for current densities of the order of 10^7 A cm^{-2} and the switching can be extremely fast (<200 ps).[16] More recently, it has been experimentally demonstrated that, under certain conditions of applied field and current density, a spin-polarized dc current induces a steady precession of the magnetization at GHz frequencies.[17–19] These steady precession effects can be obtained in $F_1/NM/F_2$ standard trilayers in which a thick magnetic layer F_1 with a fixed magnetization is used to prepare the spin-polarized current that is injected in a free thin magnetic layer F_2. The giant magnetoresistance effect[20] (GMR) of the magnetic trilayer converts the magnetic precession into microwave electrical signals. We will refer to these nonlinear oscillators as "spin transfer oscillators" (STO). They emit at frequencies which depend on field and dc current, and can present very narrow frequency linewidths.[21] As a consequence, they are promising candidates for applications in telecommunications, where the need for efficient, integrated, and frequency agile oscillators is growing. The main drawback of the spin-transfer oscillator is its very weak output microwave power that can be optimistically estimated at −40 dBm for a single oscillator. A solution to overcome this difficulty is to synchronize several oscillators, i.e., to force them to emit at a common frequency and in phase in spite of the intrinsic dispersion of their individual frequencies. This is essential for applications and this would open the way to microwave devices exploiting the fast and flexible frequency tuning of the STO by adjustment of a dc current and their unique potential for on-chip integration. On the other hand, the synchronization of STO raises complex problems which are new in spintronics and related to the general field of the dynamics of nonlinear systems.

Synchronization has been extensively studied since the 1980s, not only because of its many potential applications (in physics, biology, and chemistry) but also because understanding the behavior of a large collection of nonlinear dynamic systems is a theoretical challenge.[22,23] In solid state physics, a well-known example of synchronization is given by a network of Josephson junctions. An alternating potential takes place across a single superconductor/insulator/superconductor junction if a dc current exceeding a critical current is injected through it. For an array of such junctions, electrically connected in series or in parallel, each junction emits a microwave current that adds to the injected dc current. When the resulting interaction exceeds a critical level, it tends to synchronize the oscillation of the junctions.[24–26] The theoretical prediction[27] is that for N oscillators, not only the emitted power increases as N^2, but the frequency linewidth decreases as N^{-2}. There is a definite similarity between networks of Josephson junctions and of STO, in spite of the different equations ruling these two systems. Recent experiments have shown that STO can phase lock (synchronize) to an external microwave current source.[28] Slavin *et al.* have analytically studied this case for weakly nonlinear spin-transfer oscillators.[29] Even more recently, it has been shown experimentally that two STO can be synchronized and phase locked.[30,31] In these experiments the synchronization is supposed to be due to the coupling between the two magnetic oscillations generated in the same ferromagnetic layer by the two STO. As we will show below, it exists another way that should lead to an efficient and convenient synchronization of a large number of STO. This is by the ac current components generated by a collection of STO electrically connected in series. Nanowires composed of several hundreds of NiFe/Cu/NiFe trilayers (in series with a separation between trilayers by much thicker Cu layers) have been already fabricated by electrodeposition into holes and have been used to obtain large CPP-GMR effects.[32] Such nanowires, for which the GMR ratio can reach 30% (Ref. 32) should be ideal to implement a system of electrically coupled STO.

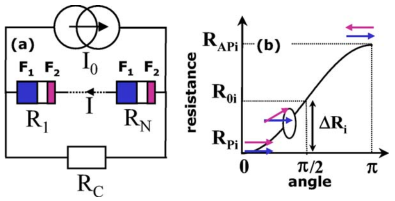

FIG. 1. (Color online) (a) Sketch of N oscillators connected in series and coupled to a load R_C (throughout the paper, R_C=50 Ω). (b) Variation of the resistance versus the angle θ_i between the magnetizations of F_1 and F_2 for an oscillator and corresponding notations.

In this paper, we develop numerical simulations to study the synchronization of electrically connected STO. More specifically, for spin-transfer oscillators electrically connected in series like the trilayers in the nanowires of Ref. 32, we introduce the coupling due to the microwave current induced in each oscillator by the oscillations of all the others. We show that, under certain conditions for the dispersion of the frequencies, the GMR amplitude and the delay between the magnetic precession and the current oscillation, synchronization can be obtained with an output power increasing as N^2 for a collection of N oscillators.

We first consider N oscillators of standard structure for spin transfer F_1(fixed)/NM/F_2(free) connected in series and coupled to a dc current generator and to a resistive load R_C, as shown in Fig. 1(a). Our notation is displayed in Fig. 1(b). We call R_{Pi} and R_{APi} the resistances of the oscillator i in, respectively, its parallel and antiparallel magnetic configurations. We define $R_{0i}=(R_{APi}+R_{Pi})/2$, $\Delta R_i=(R_{APi}-R_{Pi})/2$, $\beta_R=R_C/(R_C+\Sigma_{i=1}^N R_{0i})$, and $\beta_{\Delta Ri}=\Delta R_i/(R_C+\Sigma_{i=1}^N R_{0i})$. For the dependence of the resistance R_i of the oscillator i on the angle between the magnetizations of F_1 and F_2 at time t, $\theta_i(t)$, we assume the following standard equation:

$$R_i = R_{0i} - \Delta R_i \cos[\theta_i(t)]. \tag{1}$$

The angle $\theta_i(t)$ depends on the initial value of θ_i at t=0 and on the variation of the current between 0 and t.

In first order of $\Sigma\beta_{\Delta Ri}$ and with the notations of Fig. 1(a), a straightforward calculation leads to

$$I = I_1 + \sum_{i=1}^{N} I_1 \beta_{\Delta Ri} \cos[\theta_i(t)], \tag{2}$$

with $I_1=\beta_R I_0$. Similar expressions can be found for oscillators connected in parallel, with different expressions for J and $\beta_{\Delta Ri}$.

In order to study the behavior of N electrically coupled oscillators, we have performed simulations of the motion of the magnetizations m_j of the layers F_2 of a collection of different oscillators connected in series. Each $\mathbf{m}_j$ is considered as a macrospin without any dipolar interaction with the other $\mathbf{m}_i$. Its time evolution is given by a Landau-Lifschitz-Gilbert (LLG) equation which includes a standard spin-transfer term proportional to the current. According to Eq. (2), the current is the sum of the dc current I_1 plus the coupling term $\Sigma I_1 \beta_{\Delta Ri}\cos[\theta_i(t)]$ and the motion equation of $\hat{m}_j$ can be written as

$$\frac{d\hat{m}_j}{dt} = -\gamma_0 \hat{m}_j \times \vec{H}_{eff} + \alpha \hat{m}_j \times \frac{d\hat{m}_j}{dt} + \gamma_0 J\left[1 + \sum_{i=1}^{N} \beta_{\Delta Ri}\cos[\theta_i(t)]\hat{m}_j \times (\hat{m}_j \times \hat{M})\right], \tag{3}$$

where we have introduced the spin-transfer parameter J proportional to I and expressed it in field units. In a typical Co/Cu/Co device,[17] a current density of 10^7 A/cm^2 corresponds to about 10^{-2} Tesla. **M** is the fixed magnetization of all the F_1 layers. The effective magnetic field H_{eff}, is composed of an uniaxial anisotropy field H_{an}, an applied magnetic field H_{app}, and the demagnetizing field H_d. All fields are in-plane (parallel to the direction of the fixed magnetization of F_1) except for the out-of-plane demagnetizing field. In the following, if not mentioned otherwise, we will consider the case of 10 oscillators with H_{app}=0.2 T, H_d=1.7 T, and a Gilbert damping term α=0.007 (values for Co).

Simulations of the dynamics are performed using a fourth-order Runge-Kutta algorithm, with a calculation step of 0.5 ps. We have chosen the following random initial conditions: for each oscillator, the initial angles between the two magnetizations were randomly picked between 0° and 10° for the polar angle θ_i and between 0° and 360° for the azimuthal angle ϕ_i. We have checked that, under these conditions, the variation of the initial conditions does not hinder synchronization. In order to introduce a dispersion in the behavior of the oscillators, differences can be introduced in the anisotropy fields H_{an}, demagnetizing fields H_d or GMR ratios. We have checked that all these different types of dispersion give similar results. In this paper, we will focus on the first case, with the following dispersion: $H_{an}=0.05+(i-1)\times 0.01$ in Tesla, i varying between 1 and 10. Finally, in a real experimental setup, there may be a delay τ between the spin-transfer induced resistances variations and the resulting variation of the current. τ is zero for a perfect match between R_C and the impedance of the cables between the STO and R_C, or can be different from zero in other experimental situations. We first present simulation results obtained with a fixed value of τ (τ=5 ps, which corresponds to the limit where τ is much shorter than the precession period) and we will come back briefly on the general influence of τ at the end of this paper.

We will first consider that all the oscillators have the same resistance R and magnetoresistance ΔR, so that we can write the coupling term in Eq. (2) as $JA_{GMR}/N\Sigma\cos[\theta_i(t)]$ with $A_{GMR}=\Delta R/(R+R_C/N)$. In this particular case, for large N, A_{GMR} is close to the value of the GMR ratio.

In Fig. 2, we show the emitted power by the set of 10 oscillators as a function of the frequency for different coupling parameters A_{GMR}. For this set, with 0.05 T$\leq H_{an}\leq$0.14 T, the dispersion of the individual frequencies is 2.7%, of the order of the dispersion (1.25%) in recent experiments.[30] The emitted power at a given frequency is derived by Fast Fourier Transforming the electrical power

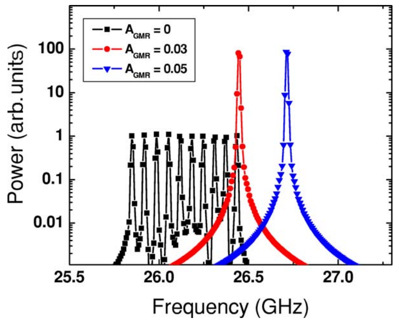

FIG. 2. (Color online) Logarithm of the power versus frequency for the set of 10 oscillators described in the text and for different coupling factors A_{GMR}. J=0.035 and τ=5 ps.

released in R_C. As the functions $\theta_i(t)$ are known only for a finite number of times t, an oscillation at a well-defined frequency appears in the Fourier transform as a peak of finite width. The injected dc current J is 0.035 T, and the delay τ=5 ps. When A_{GMR}=0, each oscillator oscillates at its own frequency: the frequencies are distributed between 25.8 and 26.5 GHz approximately, and the total emitted power is that emitted by the sum of independent oscillators. For A_{GMR} =0.03 and 0.05, all the oscillations result in a single peak. In these two cases, as expected, there is an increase by a factor of about 100 in the integrated emitted power with respect to the case without coupling. This scaling with approximately N^2 indicates that the N oscillators are locked not only in frequency but also in phase, as it will be discussed in more detail below. In Fig. 2, we can also notice a general upward shift of the frequency as the coupling increases.

In Fig. 3, we consider the evolution with time of the trajectories of 100 oscillators. The bias conditions (J=0.035 T, τ=5 ps) are similar to the previous case with H_{an} picked randomly between 0.05 and 0.1 T for each oscillator (see scale in Fig. 3), and A_{GMR}=0.03. The black curve corresponds to the two symmetrical final trajectories. By looking at the position of the oscillators at different times, we see they are turning in phase (small bounded phase shift), with the fastest oscillator opening the way.

Experimentally, varying the coupling parameter A_{GMR} means changing the GMR ratio in a controllable way, which might be difficult. Another way to increase the coupling is to increase J which, from Eq. (3), enhances both the mean

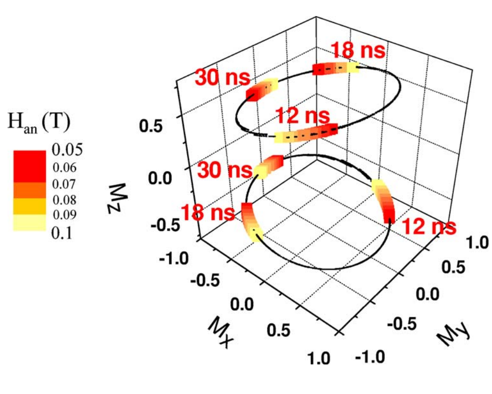

FIG. 3. (Color online) Motion of 100 oscillators on their trajectory at t=12, 18, and 30 ns: the phase of the oscillators is locked.

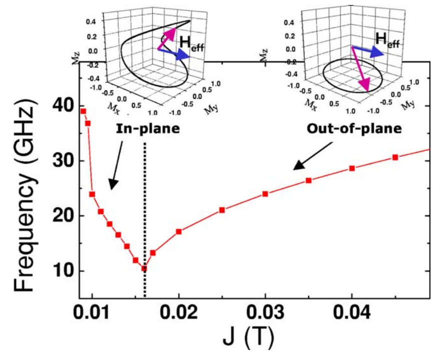

FIG. 4. (Color online) Frequency versus injected dc current J for oscillator 1 and A_{GMR}=0 (no coupling). The insets show the trajectories of the magnetization $\hat{m}$ (M_x, M_y, M_z) in the two regimes referred to in the text. In the absence of current, the equilibrium along H_{eff} corresponds to M_x=1, M_y=M_z=0.

torque and the coupling between i and j. The variation of the frequency of oscillator 1 (H_{an}=0.05 T) with J in the absence of coupling (A_{GMR}=0) is shown in Fig. 4. Similar results have been obtained in simulations by other groups.[17] For J smaller than 0.016 T, the frequency decreases as J increases in the regime of in-plane precessional trajectories of the magnetization. It increases for J larger than 0.016 T corresponding to the regime of out-of-plane orbits. In Fig. 5, we have plotted the difference in frequency, δf, between the tenth oscillator and the first as a function of J for different coupling parameters A_{GMR}; δf=0 means synchronization of the two oscillators. The reference curve (no synchronization) obtained for A_{GMR}=0 is plotted in Fig. 5(a).

We first consider the curve of Fig. 5(b) corresponding to A_{GMR}=0.03 with a delay τ of 5 ps. For low values of J, the coupling is small, and the oscillators do not synchronize. The system is nevertheless disturbed by the injection of the microwave currents, as can be seen from the differences between δf for A_{GMR}=0.03 and A_{GMR}=0. Synchronization is

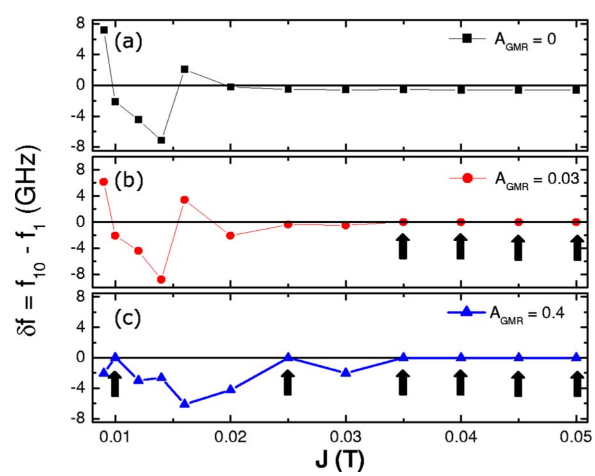

FIG. 5. (Color online) Difference in frequency between oscillator 1 (H_{an}=0.05 T) and oscillator 10 (H_{an}=0.14 T) as a function of the dc current. Three cases are considered. (a) A_{GMR}=0, (b) A_{GMR} =0.03 and τ=5 ps, (c) A_{GMR}=0.4 and τ=0.3 ns. The black arrows indicate synchronization ($f_{10}-f_1$=0).

reached [$\delta f = 0$, see arrows in Fig. 5(b)] above $J = 0.035$ T [this is in the out-of-plane regime with, as shown in Fig. 5(a), weaker dispersion, and probably, easier synchronization]. Figure 5(c) corresponds to a situation with enhanced coupling (larger A_{GMR}). In this case, synchronization extends to the in-plane precession regime (see arrow at $J = 0.01$ T).

We finally come back on the influence of the delay τ between the variation of the STO resistances and the resulting variation of the current. The results presented above are representative of the short delay limit, that is with τ much smaller than the precession period. Out of this limit, for a given set of STO and a given current, the proportion of synchronized STO depends on τ and decreases markedly when τ exceeds the precession period by about two orders of magnitude. In the intermediate range, our results also suggest that this proportion varies periodically as a function of τ with a period close to the precession period. This periodic behavior of synchronization versus delay, with a period corresponding to the oscillation frequency, has been already predicted in other systems.[33,34] This influence of the delay τ will be discussed in more detail in a further publication.

In conclusion, we have shown that it is possible to synchronize a network of spin-transfer oscillators by simply connecting them electrically in series to a load (similar effects can be expected for oscillators in parallel). The synchronization depends on the dispersion of the individual frequencies, on the coupling parameters and the delay time τ. Under certain conditions, the synchronization can be complete. In this case, the output power of N oscillators turns out to scale with N^2. We have also shown that, for synchronized oscillators, the frequency as well as the emitted power are strongly dependent on the coupling factor A_{GMR}, related to the GMR ratio. These results are of interest for obtaining an enhanced microwave generation with networks of spin-transfer oscillators. They also show that magnetic devices can be synchronized in the same way (from the coupling mechanism point of view) as in the model system represented by a network of Josephson junctions, but with two degrees of freedom (polar and azimuthal angles) instead of one (phase). As we have shown in the introduction, such series of STO could be implemented in the type of nanowires which have been developed for CPP-GMR experiments and systems of hundreds of STO could be achieved in this way.

We finally point out that the synchronization mechanism by microwave current components we have discussed for networks could also be important in the interpretation of the properties of a single spin-transfer oscillator (pillars or point contacts). The microwave spectrum of some isolated oscillators is surprisingly narrow, in contrast with the inhomogeneous broadening predicted by simulations based on micromagnetic models of ferromagnetic dots.[35] However, from our results, introducing the coupling between different parts of the dot due to the microwave component of the total current could synchronize these different parts. Such synchronization effects could thus explain less chaotic oscillations than predicted and account for the narrow linewidth of the microwave spectra.

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