Impact of the electrical connection of spin transfer nano-oscillators on their synchronization: an analytical study

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We analytically study the impact of an electrical connection of spin transfer nano-oscillators (STNOs) on their synchronization. We demonstrate that the phase dynamics of coupled STNO arrays can be described in the framework of the Kuramoto model. The conditions for successful synchronization of an assembly of STNOs are formulated. Synchronizing an assembly of STNOs appears to be the only solution to make the breakthrough on the emitted output power toward frequency synthesizers. In these potential devices, a large number of STNOs will have to be electrically connected, whatever the coupling mechanisms between oscillators. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2945636]

The spin transfer torque[1,2] in nanometer-scale magnetic devices is a consequence of the transfer of spin angular momentum from a spin-polarized current to the magnetic moment of a ferromagnet. This effect can be used to induce by injection of a dc current, some microwave steady-state magnetization precession and microwave emission in magnetoresistive devices such as spin valves or magnetic tunnel junctions.[3–8]. Due to their tunability, high frequency emission, quality factor, and high level of integration, spin transfer nano-oscillators (STNOs) are promising candidates for applications in future wireless telecommunications. Nevertheless a major breakthrough has to be performed related to their low emitted power, typically less than 1 nW. A solution is to achieve the synchronization of assemblies of STNOs, thus leading to a coherent emission and an increase in the associated power as, for example in arrays of Josephson junctions.[9] Mutual phase locking between STNOs is possible due to their intrinsic nonlinear behavior under the condition that their magnetization precessions are coupled. Local coupling mechanisms mediated by spin waves have been recently studied.[11,10] The synchronization of a single STNO to an external microwave current has also been evidenced.[12–14] These phase locking experiments are a simple approach to understand the main features of the synchronization between electrically coupled oscillators. In this vein, we have predicted by macrospin simulations that the coupling between STNOs by their self-emitted microwave currents can be large enough to achieve synchronization.[15] In this letter, we analytically determine the impact of the electrical connection of N STNOs on their synchronization coupled by their self-emitted microwave currents. In the case of connections in series or in parallel, we find the final equations to be in the frame of the Kuramoto model.[16] Finally, we discuss the resulting output power emitted by these different types of arrays.

We consider here, for simplicity, that all STNOs have the same resistance R and their precession leads to the same resistance variation $\Delta R_{\rm osc}$. Each STNO n has a phase Φ_n, that varies in time at the frequency f_0^n; and produces a microwave voltage $e_g(n) = \Delta R_{\rm osc} I_{\rm dc} \cos(\Phi_n)$, where $I_{\rm dc}$ is the dc current flowing in each STNO. We first consider the case of a series connection of N STNOs to a load Z_0, as illustrated in Fig. 1(a). The inductance and capacitance in the circuit allow to decouple the microwave from dc currents.

In order to obtain the phase dynamics of each oscillator n, we adapt the theory of weakly forced oscillators to the case of STNOs. We start from the equation for the amplitude of the spin wave mode derived by Slavin and Kabos,[17] including the spin transfer torque. From this equation of motion, the expression of the uniformly rotating phase $\Phi = \phi + N_f/\sigma I_{\rm dc} \ln(c) + \Phi_0$ of the uncoupled oscillator is derived. Here ϕ is the phase of the wave, c is its amplitude, N_f is the nonlinear frequency shift, σ is related to the spin transfer efficiency, and Φ_0 is a constant as in Ref. 13. Then we calculate the total microwave current in the loop,

$$i_{\rm hf}(\text{series}) = -\frac{\Delta R_{\rm osc} I_{\rm dc}}{Z_0 + NR} \sum_{n=1}^{N} \cos(\Phi_n). \qquad (1)$$

We add the term corresponding to $i_{\rm hf}$ in the equation of the phase dynamics of each oscillator n and assume that it acts as a weak perturbation on their limit cycle. Thus, following. Pikovski *et al.*,[18] we derive the phase dynamics of the STNO in-series array,

$$\frac{d(\Phi_n)}{dt} = -2\pi f_0^n - \frac{K}{N} \sum_{i=1}^{N} \cos(\Phi_i - \Phi_n + \Phi_0) + \xi_n(t). \qquad (2)$$

This expression is equivalent to the equation of Kuramoto *et al.*[16] The last term accounts for the Gaussian noise with

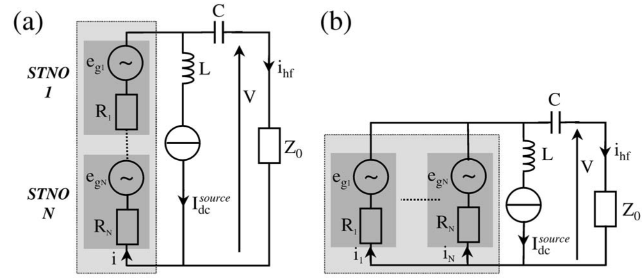

FIG. 1. Scheme for STNO connections to the load z_0 (a) in series and (b) in parallel.

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the following assumptions: $\langle\xi_n\rangle=0$ and $\langle\xi_m(t)\xi_n(t')\rangle=2w^2\delta(t-t')\delta_{mn}$ (uncorrelated in time and independent for each oscillator). The coupling factor K between in-series STNOs is expressed as

$$K_{\text{series}}=\left(\frac{\epsilon}{I_{\text{hf}}}\right)\frac{N}{Z_0+NR}\Delta R_{\text{osc}}I_{\text{dc}}, \tag{3}$$

where

$$\frac{\epsilon}{I_{\text{hf}}}=\frac{\sigma\tan(\gamma)}{2\sqrt{2}}\sqrt{\frac{I_{\text{dc}}}{I_{\text{dc}}-I_{\text{th}}}}\sqrt{1+\left(\frac{2\pi I_{\text{dc}}}{\sigma I_{\text{th}}}\frac{\partial f_0}{\partial I_{\text{dc}}}\right)^2} \tag{4}$$

is the normalized coupling strength of a single STNO to an external microwave current that was derived in our previous work.[13] The parameter γ is the equilibrium angle between the free and fixed magnetizations, I_{th} is the threshold current for the onset of oscillations, and $\partial f_0/\partial I_{\text{dc}}$ is the agility in current. We emphasize that the formula in Eq. (4) leads to a very good agreement with our phase locking results and allows us to determine experimental values of $\varepsilon/I_{\text{hf}}$.

The expression in Eq. (3) is indeed very similar to the one obtained for N Josephson junctions connected in series.[19] The crucial difference is that the resistance of superconducting Josephson junctions is extremely small. In this particular case, the coupling parameter K increases with the number of junctions. In our case of interest, the typical STNO resistance is around 10 Ω for all-metallic structures, or 200 Ω for MgO tunnel junctions. Consequently, the total resistance NR becomes larger than the typical value $Z_0=50\ \Omega$ even for a small number of oscillators. Therefore, in the case of STNOs, the coupling parameter K does not increase with N for large N, according to Eq. (3).

The phase dynamics equation [Eq. (2)] can be analytically solved, assuming that the number N of oscillators is large and that the frequency distribution is Lorentzian with a width at half maximum D^2.[16] Synchronization onset takes place when the coupling parameter K becomes larger than the critical value $K_c=2(w^2+D^2)$. This allows us to provide two important requirements for this condition to be fulfilled. The first one gives the threshold for the magnetoresistive (MR) ratio $\Delta R_{\text{osc}}/R$:

$$\left(\frac{\Delta R_{\text{osc}}}{R}\right)_{\text{series}}>\left(\frac{\Delta R_{\text{osc}}}{R}\right)_{\text{th}}=\frac{2(D^2+w^2)}{I_{\text{dc}}}\frac{1}{\left(\frac{\epsilon}{I_{\text{hf}}}\right)}. \tag{5}$$

Typical values of these parameters are 100 MHz for the frequency dispersion D^2, a linewidth w^2 of 10 MHz and $I_{\text{dc}}=5$ mA. Using 1 GHz/mA for the agility in current[20] we calculate from Eq. (4) $\varepsilon/I_{\text{hf}}=300$ MHz/mA. These values lead to a threshold ratio $\Delta R_{\text{osc}}/R$ for synchronization of about 15%. Note that $\Delta R_{\text{osc}}/R$ is not equivalent to the total MR ratio, but to the part converted in an oscillating voltage due to the precession. For example, in MgO based tunnel junctions, MR ratios as large as 100% are obtained but up to now, the largest reported power is about 50 nW, that corresponds to only $\Delta R_{\text{osc}}/R=10^{-5}$ (with $R=200\ \Omega$).[6] Moreover, in standard spin valve nanopillars, since the total MR ratio is usually lower than 10%, the first condition for synchronization would be difficult to fulfill. As already mentioned, we have predicted using macrospin numerical simulations that the synchronization could occur for MR ratios as low as 3%.[15] This discrepancy lies in the fact that much larger agilities in current (up to 10 GHz/mA) are predicted by macrospin simulations than experimentally obtained.[5]

The second requirement, expressed in Eq. (6), gives the minimum number of STNOs for the onset of synchronization.

$$N_{\text{series}}>\frac{(\Delta R_{\text{osc}}/R)_{\text{th}}Z_0/R}{\Delta R_{\text{osc}}/R-(\Delta R_{\text{osc}}/R)_{\text{th}}}. \tag{6}$$

This condition is easily fulfilled. Taking $\Delta R_{\text{osc}}/R=1.1(\Delta R_{\text{osc}}/R)_{\text{th}}$, $Z_0=50\ \Omega$, and $R=10\ \Omega$, we find $N_{\text{series}}=50$ which is commonly manufacturable.

Several routes to achieve the synchronization by the coupling via self-emitted microwave currents exist. First, a reduction in the frequency dispersion to 10 MHz, while keeping the other parameters constant, decreases the threshold for synchronization $\Delta R_{\text{osc}}/R$ down to 2.6%, that might be reached even in spin valve metallic structures. A second important improvement would be to increase the agility, for example, by achieving large angle excited modes close to the uniform mode.[5] This implies to be able to greatly reduce the device dimensions to avoid multimode excitations and also to increase the spin transfer efficiency by increasing the spin polarization and the equilibrium angle between the two magnetizations. At last, it would be necessary to increase the ratio $\Delta R_{\text{osc}}/R$, a solution being to generate large angle magnetization precessions in magnetic tunnel junctions.

The two conditions expressed in Eqs. (5) and (6) define the thresholds for synchronization in the case of a series connection. The coherent emission of all STNOs in the array will require higher coupling values.[18] Moreover, the delays in the transmission lines can hinder the coupling.[21] The analytical solutions to the extended equation of Kuramoto *et al.* with delay have been derived.[22] The best conditions for synchronization correspond to values of the delay separated by the precession period.

Note that the conditions for synchronization in the case of parallel connections are very similar to the series case. Only the number N_{parallel} is modified and it can be obtained by replacing Z_0/R by R/Z_0 in Eq. (6).

We evaluate now the achievable emitted power when all the STNOs in the array are phase locked. In that case, the microwave power delivered to the load Z_0 either for series or parallel connection is

$$P_{\text{series,parallel}}=\frac{Z_0N^2}{Z^2_{\text{series,parallel}}}\Delta R^2_{\text{osc}}I^2_{\text{dc}}, \tag{7}$$

with $Z_{\text{series}}=Z_0+NR$ and $Z_{\text{parallel}}=NZ_0+R$. Consequently, in series [see Fig. 1(a)], the power does not increase with the number of oscillators for large values of N if $NR\gg Z_0$ (this occurs when Z_0 is fixed to the standard 50 Ω). In the case of parallel connection, illustrated on Fig. 1(b), STNOs tend to shunt each other and the power will increase as N^2 only if $NZ_0\ll R$. As for the series case, a compromise must be found because reducing Z_0 leads to a decrease in the output power.

In case of on-chip applications, the value of the load Z_0 can be chosen. This offers a solution for the use of series or parallel networks by tuning Z_0 (increase in Z_0 in series, decrease in parallel). By considering in series that $Z_0=10NR$ and in parallel that $R=10Z_0N$, then if all STNOs are synchronized,

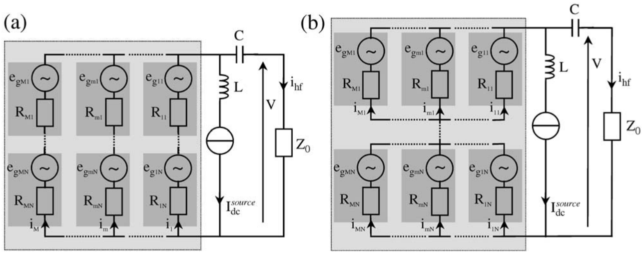

FIG. 2. Scheme for STNO connections to the load z_0 (a) hybrid 1 and (b) hybrid 2.

$$P_{\text{series}} = P_{\text{parallel}} \approx \frac{N}{10R}\Delta R_{\text{osc}}^2 I_{\text{dc}}^2. \tag{8}$$

As can be seen from Eq. (8), the power increases as N if the entire assembly is synchronized.

In order to avoid problems related to impedance matching, we propose to use "hybrid" arrays, such as the ones represented on Fig. 2. For example, in Fig. 2(a), we illustrate the case of M branches in parallel connection having each N STNOs connected in series. In Fig. 2(b), we show the case of the connection in series of N groups of M STNOs connected in parallel. In both cases, the total number of STNOs is NM. If all oscillators are synchronized, the output power of such arrays is (with I_{dc} in each branch),

$$P_{\text{hybrid}} = \frac{N^2M^2Z_0}{(NR+MZ_0)^2}\Delta R_{\text{osc}}^2 I_{\text{dc}}^2. \tag{9}$$

It is therefore enough to choose $NR=MZ_0$ to fulfill the impedance matching conditions. Then the power increases as NM. For these hybrid networks (see Fig. 2), we find that the phase of the oscillator (n,m) (n^{th} STNO of branch number m) is ruled by the following equation:

$$\frac{d(\Phi_{n,m})}{dt} = -2\pi f_0^{n,m} + \frac{K_1}{N}\sum_{i=1}^{N}\cos(\Phi_{i,m}-\Phi_{n,m}+\Phi_0) - \frac{K_2}{NM}\sum_{i=1}^{N}\sum_{j=1}^{M}\cos(\Phi_{i,j}-\Phi_{n,m}+\Phi_0) + \xi_{n,m}(t), \tag{10}$$

where

$$K_1 = \left(\frac{\epsilon}{I_{\text{hf}}}\right)\frac{\Delta R_{\text{osc}}}{R}I_{\text{dc}}, K_2 = \left(\frac{\epsilon}{I_{\text{hf}}}\right)\frac{Z_0}{R}\frac{M\Delta R_{\text{osc}}}{MZ_0+NR}I_{\text{dc}}. \tag{11}$$

Eq. (10) corresponds to an extended version of the equation of Kuramoto *et al.* that has no simple analytical solution. For large values of N (and M), the coupling parameters K_1 and K_2 are again independent on the number of oscillators. Due to the similarity with the series connection case derived in Eq. (2), we believe that the thresholds for synchronization in hybrid networks will not be very different from those given in Eqs. (5) and (6).

A general comment is that the aforementioned considerations on the emitted output power in STNOs arrays or networks remain valid for all types of coupling mechanism, i.e, through spin waves or by dipolar fields, simply because STNOs must be somehow electrically connected. We believe that the best solutions as far as the power is concerned, are networks having a mix of series and parallel electrical connection of STNOs. Note that the nanocontact geometry that is often presented as promising for synchronization through the local spin-wave coupling,[11,10] is hardly suitable for connecting STNOs in series.

In summary, we have analytically studied the synchronization effect by self-emitted microwave currents in electrically connected arrays of STNOs. In this scheme, STNOs are well described by the Kuramoto model from which the conditions for successful synchronization are derived. Using values of the coupling efficiency to a microwave current extracted from our experiments, we give the criteria for the microwave characteristics and the total number of STNOs necessary for phase locking. Moreover, we have calculated the output power when a complete synchronization is achieved. We believe that a breakthrough in the output power delivered by STNOs for the application in telecommunication can be made using the hybrid arrays we propose.

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